AMAT112: Calculus I

Antiderivatives and Rectilinear Motion

1. Find an anti-derivative for each of the following.

   a) $f(x) = x^2 - 2x + 1$ b) $g(t) = t^7 - 6t + 8$ c) $h(s) = s^{-4} + 2s + 3$

   d) $w(z) = 2 - \dfrac{5}{z^2}$ e) $v(t) = \sqrt{t} + \dfrac{1}{\sqrt{t}}$ f) $g(\theta) = \sec^2(\theta) - \sin(\theta)$

   g) $r(a) = -\csc(a)\cot(a) + \dfrac{1}{a}$ h) $f(x) = \dfrac{1}{1+x^2} - \dfrac{1}{\sqrt{1-x^2}}$ i) $v(u) = e^u + \csc^2(u) - \dfrac{1}{u^2}$

   The following two exercises are from the textbook section 4.7.

2. A stone is dropped from the upper observation deck of a tower, 405 m above the ground.

   a) Find the distance of the stone above ground level at time t.

   b) How long does it take the stone to reach the ground?

   c) With what velocity does it strike the ground?

   d) If the stone is thrown downward with a speed of 5 m/s, how long does it take to reach the ground?

3. Two balls are thrown upward from the edge of a cliff. The first is thrown with a speed of 48 ft/s and the other is thrown one second later with a speed of 24 ft/s. When do the balls pass each other?

4. A hot air balloon ascending at a rate of 12 ft/sec is at a height 80 ft above the ground when a package is dropped. How long does it take the package to reach the ground?

5. A rocket lifts off the surface of Earth with a constant acceleration of 20 m/sec^2. How fast will the rocket be going 1 minute later?

6. (a) Suppose that the velocity of a body moving along the s-axis is $\dfrac{ds}{dt} = v = 9.8t - 3$.
   i. Find the body's displacement from $t = 1$ to $t = 3$ given that $s = 5$ when $t = 0$.
   ii. Find the body's displacement from $t = 1$ to $t = 3$ given that $s = -2$ when $t = 0$.
   iii. Now find the body's displacement from $t = 1$ to $t = 3$ given that $s = s_0$ when $t = 0$.

   (b) Suppose that the position of a body moving along a coordinate line is a differentiable function of time t. Is it true that once you know an antiderivative of the velocity function ds/dt you can find the body's displacement from $t = a$ to $t = b$ even if you do not know the body's exact position at those times? Give reasons for your answer.